

UMJ-4751
B.A./B.Sc. (Fifth Semester)
Examination, 2024-25
MATHEMATICS
Paper - I (Theory)
[Abstract Algebra]

Time : 3 Hours]

[Maximum Marks : 75

Note : This question paper is divided into two sections A and B. Attempt any *five* questions each of 6 marks from the section A and any *three* questions each 15 marks from Section B.

SECTION-A
(Short Answer Type Questions)

Note : Attempt any *five* questions from this section. Each question carries equal marks. (5×6=30)

1. Show that the set $S = \{1, 5, 7, 11\}$ is a group with respect to the multiplication module 12.

2. Prove that every cyclic group is abelian. Show by an example that converse may not be true in general.
3. Let G be the group of real numbers under addition and let $\bar{G}$ be the group of positive real numbers under multiplication. Show that G and $\bar{G}$ are isomorphic under $\phi : G \rightarrow \bar{G}$, $\phi(x) = 2^x$.
4. Show that every subgroup of an abelian group is normal.
5. Let R be a commutative ring. Then prove that R is integral domain if and only if the cancellation laws hold in R .
6. Prove that the intersection of any two subrings of a ring R is a subring of R .
7. Let R be the ring of all real valued continuous functions on $[0, 1]$. Show that the set $S = \left\{ f \in R : f\left(\frac{1}{2}\right) = 0 \right\}$ is an ideal of R .
8. Define the following terms :
 - (a) Characteristics of a Ring.
 - (b) Center of the group.
 - (c) Units in a ring.

SECTION-B

(Long Answer Type Questions)

Note : Attempt any *three* questions from this sections. Each question carries equal marks. (3×15=45)

9. Show that the collection A_3 of even permutation forms a subgroup of the symmetric group S_3 with respect to the permutation multiplication. Check whether A_3 is a normal subgroup of S_3 .

10. Prove that, for all element a and b of the group G

$$(ab)^{-1} = b^{-1}a^{-1}.$$

11. State and prove Lagrange's theorem.

12. Show that the set

$$\mathbb{Z}[i] = \{a + ib : a, b \in \mathbb{Z}\}$$

forms a ring under ordinary addition and multiplication of complex numbers, where $i = \sqrt{-1}$. Is $\mathbb{Z}[i]$ an integral domain?

13. Prove that a finite integral domain is a field.

B.A./B.Sc. (Fifth Semester)**Examination, 2025-26****MATHEMATICS****Paper – I (Major)****[Abstract Algebra]****Time : 3 Hours]****[Maximum Marks : 75**

Note : The question paper is divided into two sections 'A' and 'B'. Attempt any *five* questions from Section 'A' and *three* from Section 'B'. Marks are indicated against each section.

SECTION-A**(Short Answer Type Questions)**

Note : Attempt any *five* questions from the following *eight* questions. Each question carries 6 marks. ($5 \times 6 = 30$)

1. Let $H = \{(1), (12)\}$. Is H normal in S_3 ?

2. Show that $U(8)$ is not isomorphic to $U(10)$ and isomorphic to $U(12)$.
3. How many homomorphisms are there from $\mathbb{Z}_{20}$ onto $\mathbb{Z}_8$ and $\mathbb{Z}_{20}$ onto $\mathbb{Z}_4$?
4. Find all automorphism of the group $\mathbb{Z}_{12}$.
5. Find all the Sylow 3-subgroups of S_4 .
6. Find all maximum ideals in :
 - (a) $\mathbb{Z}_8$
 - (b) $\mathbb{Z}_{10}$
 - (c) $\mathbb{Z}_{12}$
7. Show that every subgroup of an abelian group is normal. ✓
8. Show that the set $S = \{1, 5, 7, 11\}$ is a group with respect to the multiplication modulo 12. (Full) ✓

SECTION-B

(Long Answer Type Questions)

Note : Attempt any *three* questions from the following *five* questions. Each question carries 15 marks.

(3×15=45)

9. Let

$$\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{bmatrix} \text{ and } \beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{bmatrix}$$

Compute each of the following :

(a) α^{-1} .

(b) $\beta\alpha$ *value*

(c) $\alpha\beta$ *value*

10. Prove that a group G is abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}$ for all a and b in G .

11. Prove that the following rings are integral domains :

(a) The ring of integers.

(b) The ring of Gaussian integer $Z[i] = \{a + bi | a, b \in Z\}$

(c) The ring $Z[\sqrt{2}] = \{a + b\sqrt{2} | a, b \in Z\}$.

12. Prove that :

(a) Every field is an integral domain.

(b) Finite integral domain is field.

13. State and prove Lagrange's theorem.