

Examination, 2024-25

MATHEMATICS

Paper - II (Theory)

[Linear Algebra]

Time : 3 Hours]

[Maximum Marks : 75

Note : This question paper contains *two* sections A and B. Attempt any *five* questions each of 6 marks from Section A and any *three* questions each 15 marks from Section B.

SECTION-A

(5×6=30)

1. Let S be a non empty set and F a field. Prove that for any $s_0 \in S$, $\{f \in \mathcal{F}(S, F) : f(s_0) = 0\}$, is a subspace of $\mathcal{F}(S, F)$.
2. Let $S = \{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$ be a subset of the vector space $\mathbb{R}^3$. Then show that S is linearly independent.

3. Show that the set $S = \{(1, 0, -1), (2, 5, 1), (0, -4, 3)\}$ is basic for $\mathbb{R}^3$.
4. If $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ defined by

$$T(a_1, a_2) = (a_1 + a_2, 0, 2a_1 - a_2)$$
 Then prove that T is a linear Transformation.
5. Let T be a linear operator on V , such that $V = \mathbb{R}^2$ and $T(a, b) = (-2a + 3b, -10a + 9b)$. Find the eigen values of T and an ordered bases β for V such that $[T]_\beta$ is a diagonal matrix.
6. Explain invariant subspaces.
7. Let V be an inner product space over F . Then all $x, y \in V$,

$$|\langle x, y \rangle| \leq \|x\| \cdot \|y\|.$$
8. Define :
 - (a) Diagonalizable Operator.
 - (b) Annihilators.

SECTION-B

(3×15=45)

9. Let w_1 and w_2 be subspaces of a vector space V , then
 - (a) Prove that $w_1 + w_2$ is a subspace of V that contains both w_1 and w_2 .

(b) Prove that any subspace of V that contains both w_1 and w_2 must also contain $w_1 + w_2$.

10. Let V and W be vector space, and let $T : V \rightarrow W$ be linear. If V is finite dimensional, then

$$\text{nullity}(T) + \text{rank}(T) = \dim(V).$$

11. State and prove the Cayley-Hamilton Theorem.

12. Prove that the restriction of a linear operator T to a T -invariant subspace is a linear operator on that subspace.

13. Apply the Gram-Schmidt process to the given subset S of the inner product space V to obtain an orthogonal basis for $\text{span}(S)$, where $V = \mathbb{R}^3$, $S = \{(1, 0, 1), (0, 1, 1), (1, 3, 3)\}$.

2024/12/29

B.Sc. (Fifth Semester) (NEP)

Examination, 2025-26

MATHEMATICS

Paper – II (Major)

[Linear Algebra]

Time : 3 Hours]

[Maximum Marks : 75

Note : The question paper is divided into *two* sections 'A' and 'B'. Attempt any *five* questions from Section 'A' and *three* from Section 'B'. Marks are indicated against each section.

SECTION–A

(Short Answer Type Questions)

Note : Attempt any *five* questions from the following *eight* questions. Each question carries 6 marks. ($5 \times 6 = 30$)

1. Calculate the number of elements in a vector space of polynomials of degree at most n . Here coefficients are the elements of the field $\mathbb{F}(p)$, where p is a prime number.

[P.T.O.]

2. If a_1, a_2, a_3 are fixed elements of a field F , then prove that the set W of all ordered triads (x_1, x_2, x_3) of elements of F such that $a_1x_1 + a_2x_2 + a_3x_3 = 0$; is a subspace of $V_3(F)$.
3. Show that the three vectors $(1, 1, -1)$, $(2, -3, 5)$ and $(-2, 1, 4)$ of $\mathbb{R}^3$ are linearly independent.
4. Describe explicitly the linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ such that $T(2, 3) = (4, 5)$ and $T(1, 0) = (0, 0)$.
5. Find the dual basis of the basis set
 $B = \{(1, -1, 3), (0, 1, -1), (0, 3, -2)\}$ for $V_3(\mathbb{R})$.
6. Define inner product space with a suitable example.
7. State and prove Cauchy-Schwartz inequality.
8. Write a short note on various classes of a real quadratic form.

SECTION-B

(Long Answer Type Questions)

Note : Attempt any *three* questions from the following *five* questions. Each question carries 15 marks.

(3×15=45)

9. Prove that a subset W of a vector space $V(\mathbb{F})$ is a subspace of $V(\mathbb{F})$ if and only if $a\alpha + b\beta \in W \forall a, b \in \mathbb{F}$ and $\forall \alpha, \beta \in W$.
10. If W_1 and W_2 are subspaces of a finite dimensional vector space $V(\mathbb{F})$, then prove that :
- $$\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$$
11. Prove that the vectors $\alpha_1 = (1, 0, -1)$, $\alpha_2 = (1, 2, 1)$, $\alpha_3 = (0, -3, 2)$ form a basis for $\mathbb{R}^3$. Express each of the standard basis vectors as a linear combination of α_1 , α_2 and α_3 .
12. State and prove Cayley-Hamilton theorem.
13. Let U and V be vector spaces over the field F and let T be a linear transformation for U into V . Suppose that U is finite dimensional. Prove that :

$$\text{rank } (T) + \text{nullity } (T) = \dim U.$$
