

UMJ-4741
B.Sc. IV Semester (NEP)
Examination, 2024
MATHEMATICS
Paper-I (Theory)
[Differential Equations]

Time : 3 Hours]

[Maximum Marks : 75

Note : This question paper contains *two* sections A and B. Attempt any *five* questions each of 6 marks from Section A and any *three* questions each 15 marks from Section B.

SECTION-A

(5×6=30)

1. Solve $(2x + y + 1)dx + (4x + 2y - 1)dy = 0$.
2. Solve $(x + 2y^3)dy = y dx$.
3. Find general and singular solution of the differential equation $(xp - y)^2 = p^2 - 1$.

4. Solve $(D^2 + D - 6)y = e^{2x}$.

5. Solve $\frac{d^2 y}{dx^2} + 2 \tan x \cdot \frac{dy}{dx} + 3y = \tan^2 x \sec x$.

6. Find the complete integral of $z^2(p^2 z^2 + q^2) = 1$.

7. Classify the following partial differential equation of second

order : $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$.

8. Solve using Laplace transforms :

$\frac{d^2 y}{dt^2} + y = t, \quad y(0) = 1, \quad \left. \frac{dy}{dt} \right|_{t=0} = -2$.

SECTION-B

(3×15=45)

9. (a) Solve the following partial differential equation :
 $2r - s - 3t = 5e^{x-y}$.

(b) Solve $s + p - q = z + xy$.

10. (a) Verify that the equation :

$(y-z)(y+z-2x)dx + (z-x)(z+x-2y)dy + (x-y)(x+y-2z)dz = 0$ is exact and find the solution.

(b) Solve $\frac{dx}{y^2 + z^2 - x^2} = \frac{dy}{-2xy} = \frac{dz}{-2xz}$.

11. Find the series solution of the equation $x \frac{d^2 y}{dx^2} + \frac{dy}{dx} + xy = 0$.

12. (a) Find the orthogonal trajectories of the family curves

$\frac{x^2}{(a^2 + \lambda)} + \frac{y^2}{(b^2 + \lambda)} = 1$, where λ is the parameter.

(b) Solve $\frac{x dx + y dy}{x dy - y dx} = \sqrt{\frac{a^2 - x^2 - y^2}{x^2 + y^2}}$.

13. (a) Find $L(e^{-t} \sin^2 t)$, L denote Laplace transform.

(b) Find $L^{-1}\left(\frac{6s-4}{s^2-4s+20}\right)$, L^{-1} denote the inverse Laplace transform.