UPC - 001321128801

Total Pages : 4 Roll No.

B.A./B.Sc. II Semester Examination, 2025

MATHEMATICS

Paper–I

[Geometry]

Time : 3 Hours] **[Maximum Marks : 75**

Note : There are *two* sections A and B in this paper. In Section A attempt any *five* questions and from Section B attempt any *three* questions. Marks are indicated against each section.

SECTION–A

(Short Answer Type Questions)

Note : Attempt any *five* questions from the following eight questions. Each question carries 6 marks. (5×6=30)

1. (a) Find the polar equation of a conic with its focus as the pole and its axis inclined at an angle α to the inital line.

(b) In a conic prove the sum of the reciprocals of the segments of any focal chord is constant i.e.

$$\frac{1}{SP}+\frac{1}{SP'}=\frac{2}{l} \text{ (constant).}$$

2. Find the equation of the plane passing throught the point (1, 2, 1) and perpendicular to the line joining the points (1, 4, 2) and (2, 3, 5).

3. Find the distance of the point (2, 3, 4) from the point where the line $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}$ meets the plane $x + y + z = 22$.

4. If l, m, n are the direction cosines of a line, then prove that $l^2 + m^2 + n^2 = 1$.

5. Find the equation of a sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, $2x + 3y + 4z = 8$ is a great circle.

6. The vertex of a cone is (a, b, c) and the yz-plane cuts it in the curve $F(x, y) = 0$, $x = 0$, show that the zx-plane cuts it in the curve.

7. Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1}=\frac{y}{-2}=\frac{z}{3}$ and whose guiding curve is the ellipse $x^2 + 2y^2 = 1$, $z = 3$.

8. Find the condition that the plane $lx + my + nz = p$ may touch the central conicoid $ax^2 + by^2 + cz^2 = 1$.

SECTION–B

(Long Answer Type Questions)

Note : Attempt any *three* questions. Each question carries 15 marks. (3×15=45)

9. (a) Find the Intercepts form of the equation of a plane.

(b) Find equation of plane throught the point (1, 3, 2) and parrallel to the plane $3x - 2y + 2z + 33 = 0$.

10. (a) Find shortest distance between :

$$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}, \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$$

(b) Find symmetrical form of the line $3x + 2y - z - 4 = 0$, $4x + y - 2z + 3 = 0$.

11. (a) Find the equation of the tangent at the point (r_1, θ_1) of the conic $\frac{l}{r} = 1 + e\cos\theta$.

(b) Find equation of spheres which pass through the circle $x^2 + y^2 + z^2 = 5$, $x + 2y + 3z = 3$ and touch the plane $4x + 3y = 15$.

12. (a) Prove that the equation :

$4x^2 - y^2 + 2z^2 + 2xy - 3yz + 12x - 11y + 6z + 4 = 0$ represents a cone and find the co-ordinates of its vertex.

(b) Find the equation to the plane which cuts $3x^2 + 2y^2 - 15z^2 = 4$ in a conic whose centre is at the point $(-2, 3, -1)$.

13. (a) Find equation of the right circular cylinder whose radius is 'r' and axis the line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-r}{n}$.

(b) If θ is the angle between two lines or vectors whose direction cosines are l_1, m_1, n_1 and l_2, m_2, n_2 then prove that $\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$.
