

UMJ-4731

B.A./B.Sc. (Third Semester)

NEP - Examination, 2023-24

MATHEMATICS

[Paper- First (Major)]

[Calculus]

[Time : 3 Hours]

[Maximum Marks : 75]

Note: This question paper is divided into two section A and B. Attempt any five questions from Section A and any three questions from Section B. Marks are indicated against each section.

SECTION - A
(Short Answer Type Questions)

Note: Attempt any five questions from the following 8 questions. Each question carries 6 marks.

(5×6=30)

1. ✓ If, function

$$f(x) = \begin{cases} \frac{\sin[x]}{[x]} & , [x] \neq 0 \\ 0 & , [x] = 0 \end{cases}$$

Where $[x]$ denotes the greatest integer less than or equal to x , then find $\lim_{x \rightarrow 0} f(x)$.

2. ✓ Discuss the continuity of the function defined as follows:

$$(i) \quad f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & ; x \neq 0 \\ 2 & ; x = 0 \end{cases}$$

$$(ii) \quad f(x) = \begin{cases} \frac{e^{\sqrt{x}} - 1}{e^{\sqrt{x}} + 1} & ; x \neq 0 \\ 0 & ; x = 0 \end{cases}$$

3. ✓ Show that the function of given by $f(x) = x \tan^{-1}\left(\frac{1}{x}\right)$ for $x \neq 0$ and $f(0)=0$ is continuous but not differentiable at $x=0$.

4. ✓ Discuss applicability of Rolle's Theorem to the function $f(x)=|x|$ in $[-1, 1]$.

5. ✓ If the normal to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ make an angle ϕ with the axis of x , show that its normal equation is $y \cos \phi - x \sin \phi = a \cos 2\phi$.

6. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 u = \frac{-9}{(x+y+z)^2}$$

7. (a) Evaluate $\int_0^{\pi} \frac{x \sin x}{1 + \cos 2x} dx$

- (b) Evaluate $\iint xy(x+y) dx dy$ over the area between $y=x^2$ and $y=x$.

8. Expand $\sin^{-1}(x+a)$ in power of x as far as the term in x^3 .

SECTION - B (Long Answer Type Questions)

Note: Attempt any three questions from the following 5 questions. Each question carries 15 marks.

(3×15=45)

9. (a) Explain $\log_e(1+\sin x)$ by Maclaurin's theorem.
(b) Find the radius of curvature of the curve $y=e^x$ at the point where it crosses the y-axis.
10. (a) Find $(y_n)_0$, when $y=e^{m \sin^{-1} x}$
(b) Find the points of inflexion of the curve $y(a^2+x^2)=x^3$

11. (a) Trace the curve $r = a \cos 3\theta$.

(b) Show that $\int_0^{\pi/2} \tan^n \theta d\theta = \frac{1}{2} \pi \sec\left(\frac{1}{2} n \pi\right)$.

✓ 12. Find the volume of the solid generated by the revolution of the cissoid $y^2(2a-x) = x^3$ about its asymptote.

13. (a) ✓ Prove that the spirals $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$ intersect orthogonally.

✓ (b) If $x+y+z = u$, $y+z = uv$, $z = uvw$ find the value of the Jacobian of x, y, z with respect to u, v, w .

Total Pages : 4

Roll No.

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Paper - First(Major)
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Time : 3 Hours]**[Maximum Marks : 75**

Note : This question paper is divided into *two* sections A and B. Attempt any *five* questions from Section A and any *three* questions from Section B. Marks are indicated against each section.

SECTION-A
(Short Answer Type Questions)

Note : Attempt any *five* questions from the following eight questions. Each question carries 6 marks. (5×6=30)

1. Prove that if $f(x) \leq M$ for all x in some open interval containing a and if $\lim_{x \rightarrow a} f(x) = A$, then $A \leq M$.

2. Discuss the continuity of the functions defined as follows :

(a) $f(x) = |x| - x, x \in \mathbb{R}.$

(b) $f(x) = \frac{x^4 - 1}{x^2 - 1}, x \neq 0 \text{ and } f(0) = 1.$

3. Prove that continuity is a necessary but not sufficient condition for differentiability.

4. Discuss the applicability of Rolle's theorem to the function defined by

$$f(x) = \frac{x^2 - 3x - 4}{(x - 5)} \text{ in } [-1, 4].$$

5. Find equations of the tangent and normal lines to the curve $x^2 - y^2 = 7$ at the point $(4, -3).$

6. If $u = f(x - y, y - z, z - x),$ then show that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0.$$

7. Evaluate by summation $\int_a^b \frac{1}{x^2} dx.$

8. Evaluate : $\int_0^{\infty} e^{-4x} x^{5/2} dx.$

SECTION-B

(Long Answer Type Questions)

Note : Attempt any *three* questions from the following 5 questions. Each questions carry 15 marks. (3×15=45)

9. (a) Prove that the radius of curvature at any point 't' of the cycloid $x = a(t + \sin t)$, $y = a(1 - \cos t)$ is

$$p = 4a \cos\left(\frac{t}{2}\right).$$

- (b) Expand $e^{x \sin x}$ by Maclaurin's Theorem.

10. (a) Find $(y_n)_0$ when $y = \sin^{-1} x$.

- (b) Find all the esymptotes of the curve

$$(x + y)^2(x + 2y + 2) = x + 9y - 2.$$

11. (a) Trace the curve $y^2(2a - x) = x^3$.

- (b) Prove that $\int_0^{\pi/2} \sin^{(2p-1)} \theta \cos^{(2q-1)} \theta d\theta = \frac{1}{2} \beta(p.q).$

12. (a) If $x + y + z = u$, $y + z = uv$ and $z = uvw$, find the value of Jacobian of x, y, z with respect to u, v, w .

(b) Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x dy dx dz$.

13. (a) Find the volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about x-axis.

- (b) Find the perimeter of the cardioid $r = a(1 - \cos\theta)$.
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